

AXIAL GAUGE IN EUCLIDEAN QUANTUM GRAVITY

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Abstract. The axial gauge is applied to the analysis of Euclidean quantum gravity on manifolds with boundary. A set of boundary conditions which are completely invariant under infinitesimal diffeomorphisms require that spatial components of metric perturbations should vanish at the boundary, jointly with all components of the ghost one-form and of the gauge-averaging functional. If the latter is taken to be of the axial type, all components of metric perturbations obey Dirichlet conditions, and all ghost modes are forced to vanish identically. The one-loop divergence coincides with the contribution resulting from three-dimensional transverse-traceless perturbations.

Axial Gauge in Euclidean Quantum Gravity

The recent attempts to obtain a correct formulation of Euclidean quantum gravity on manifolds with boundary [1] have shed new light on the quantization programme in covariant or non-covariant gauges. The consideration of boundaries is suggested by the problems of quantum field theory (e.g. Casimir effect, van der Waals forces) and by the attempts to define the quantum state of the universe [1]. Whenever boundaries occur, one would like to ensure that the whole set of boundary conditions are invariant under infinitesimal gauge transformations, since the underlying classical theory has this property, and perturbative quantum theory may be viewed as a theory of small disturbances around some background-field configurations. In the case of the gravitational field, which is the object of our investigation, the whole set of metric perturbations h_{ab} are subject to the infinitesimal gauge transformations ${}^\varphi h_{ab} \equiv h_{ab} + \nabla_{(a} \varphi_{b)}$, where ∇ is the Levi-Civita connection of the background four-geometry with metric g , and $\varphi_\nu dx^\nu$ is the ghost one-form. For problems with boundaries, one thus finds

$${}^\varphi h_{ij} = h_{ij} + \varphi_{(i|j)} + K_{ij}\varphi_0 , \quad (1)$$

where the stroke denotes three-dimensional covariant differentiation tangentially with respect to the intrinsic Levi-Civita connection of the boundary, whilst K_{ij} is the extrinsic-curvature tensor of the boundary (we assume that K_{ij} is nowhere vanishing). By virtue of (1), the boundary conditions

$$\left[h_{ij} \right]_{\partial M} = 0 \quad (2)$$

are also obeyed by ${}^\varphi h_{ij}$ if and only if the whole ghost one-form obeys homogeneous Dirichlet conditions:

$$\left[\varphi_a \right]_{\partial M} = 0 , \quad \forall a = 0, 1, 2, 3 . \quad (3)$$

The problem now arises to impose boundary conditions on the remaining set of metric perturbations, in such a way that their invariance under infinitesimal diffeomorphisms is again guaranteed by (3), since otherwise one would obtain incompatible sets of boundary conditions on the ghost one-form. For this purpose, one can point out that, if Φ_a is any gauge-averaging functional which leads to self-adjoint elliptic operators on metric perturbations, one finds

$$\delta\Phi_a(h) \equiv \Phi_a(h) - \Phi_a({}^\varphi h) = \mathcal{F}_a{}^b \varphi_b , \quad (4)$$

where $\mathcal{F}_a{}^b$ is an elliptic operator that acts linearly on the ghost one-form. Thus, if one imposes the boundary conditions

$$\left[\Phi_a(h) \right]_{\partial M} = 0 , \quad \forall a = 0, 1, 2, 3 , \quad (5)$$

their gauge invariance is guaranteed when (3) holds, by virtue of (4). Hence one also has $\left[\Phi_a(\varphi h)\right]_{\partial M} = 0, \forall a = 0, 1, 2, 3$.

If the axial (A) gauge-averaging functional is used: $\Phi_a^{(A)}(h) \equiv n^b h_{ab}$, a considerable simplification of the boundary conditions is obtained, since all h_{ab} perturbations are then set to zero at ∂M . The resulting ghost operator takes the form [2]

$$\mathcal{F}_a{}^b = 2\delta_a^{(b} n^{c)} \nabla_c. \quad (6)$$

This implies that the ghost operator does not have any eigenfunctions at all. Indeed, given the eigenvalue equation $\mathcal{F}\varphi_\lambda = \lambda\varphi_\lambda$, its solution in the coordinates τ, \hat{x} (τ being a radial coordinate, and \hat{x} local coordinates on ∂M) is [2]

$$\varphi_{0_\lambda}(\tau, \hat{x}) = \exp\left(\frac{1}{2}\lambda\tau\right) f_{0_\lambda}(\hat{x}), \quad (7)$$

$$\begin{aligned} \varphi_{i_\lambda}(\tau, \hat{x}) &= \exp(\lambda\tau) g_{ij}(\tau, \hat{x}) f_\lambda^j(\hat{x}) \\ &- \int_0^\tau dy \exp\left[\lambda\left(\tau - \frac{1}{2}y\right)\right] g_{ij}(\tau, \hat{x}) g^{jk}(y, \hat{x}) \widehat{\nabla}_k f_{0_\lambda}(\hat{x}). \end{aligned} \quad (8)$$

Now imposing the boundary conditions (3) one finds $f_{0_\lambda} = f_\lambda^i = 0$, and hence $\varphi_\lambda = 0$ for any λ . Thus, ghost fields do not contribute at all to the transition amplitudes in our problem.

The remaining part of the analysis in the axial gauge is as follows [2]. The spectrum of the operator on metric perturbations can be obtained by studying the spectrum of the operator (here $\square \equiv g^{ab}\nabla_a\nabla_b$)

$$\begin{aligned} \Delta^{ab,cd} &= -\left(g^{a(c} g^{d)b} - g^{ab} g^{cd}\right) \square - g^{cd} \nabla^{(a} \nabla^{b)} \\ &- g^{ab} \nabla^{(c} \nabla^{d)} + 2\nabla^{(a} g^{b)(c} \nabla^{d)}. \end{aligned} \quad (9)$$

Covariant differentiation of the resulting eigenvalue equation, multiplication by g^{ab} with summation over repeated indices, and imposition of the axial gauge imply that the only non-vanishing metric perturbations are the three-dimensional transverse-traceless tensors. This remains true if one performs a Gaussian average over the axial-type functional. The integrability condition for the eigenvalue equation contains then further contributions resulting from the extrinsic-curvature tensor, and selects three-dimensional transverse-traceless perturbations, provided that the unperturbed extrinsic-curvature tensor is proportional

to the three-metric of the boundary [2]. The full one-loop divergence is then given by $\zeta(0) = -\frac{278}{45}$, if one studies a portion of flat Euclidean four-space bounded by a three-sphere [2]. This is a non-trivial property, since a gauge has been found such that the contributions of ghost and gauge modes vanish separately in the presence of boundaries. This property is not shared by other non-covariant gauges, e.g. the Coulomb gauge for Euclidean Maxwell theory, where the ghost and gauge contributions cannot be made to vanish separately for problems with boundary [2].

The axial gauge has been recently applied to the one-loop semiclassical analysis of simple supergravity in the presence of boundaries [1,3]. This analysis shows that the effects of three-dimensional transverse-traceless perturbations for gravitons and gravitinos do not cancel each other exactly, and hence simple supergravity is one-loop finite just in those particular backgrounds with boundary where pure gravity is one-loop finite as well [4].

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